approximation theory which furnishes valuable insight into 'who considers what worthwhile and interesting'.

J. R. R.

20 [2.05.2].—R. P. FEINERMAN & D. J. NEWMAN, *Polynomial Approximation,* The Williams & Wilkins Co., Baltimore, Md., 1974, viii + 148 pp., 24 cm. Price $13.00.

A descriptive title for this book is "Degree of convergence for polynomial and rational approximation on the real line". This is a thorough and compact presentation of most of the known theory on this topic, the primary exclusions being those results that involve complex functions, analyticity, etc. There is a short (ten pages) chapter on the existence, uniqueness and characterization of best Tchebycheff approximations; and, otherwise, there is very little that does not relate directly to degree of convergence questions. Thus the scope of the book is rather narrow and it is not suitable as a general reference or text on approximation theory (even polynomial approximation).

As a special topics book, it is well done. The authors have organized the material well and concisely. There is a natural progression from traditional results to current research (to which one of the authors is a principal contributor) which the specialist in approximations theory will find readable and interesting. There are only thirty-eight items in the bibliography. The book is done economically as far as design, copy-editing and production are concerned; and only one misprint was noted (reference [25]).

J. R. R.

21 [2.05, 7].— HERBERT E. SALZER, *Laplace Transforms of Osculatory Interpolation Coefficients,* ozalid copy of handwritten ms. of six sheets, 11" × 16", deposited in the UMT file.

The Laplace transforms of the n-point $(2n - 1)$th-degree osculatory interpolation coefficients based on the integral points $i = 0(1)n - 1$, namely,

$$A_i^{(n)}(p) = \int_0^\infty e^{-pt}\{[L_i^{(n)}(t)]^2\,[1 - 2L_i^{(n)\prime}(i)(t - i)]\}\,dt,$$

$$B_i^{(n)}(p) = \int_0^\infty e^{-pt}\{[L_i^{(n)}(t)]^2\,(t - i)\}\,dt,$$

where

$$L_i^{(n)}(t) = \prod_{j=0,j\neq i}^{n-1} (t - j) \Big/ \prod_{j=0,j\neq i}^{n-1} (i - j),$$

are expressed exactly as functions of p, for $n = 2(1)9$. Both $A_i^{(n)}(p)$ and $B_i^{(n)}(p)$ underwent three functional checks that were made on the exact fractional coefficients of p^{-r}, $r = 1(1)2n$, on the final manuscript. All computations were performed with a desk calculator before 1962, except for the recent completion of the final checks by hand.

Given $f(i)$ and $f'(i)$, $i = 0(1)n - 1$, we have the approximation

$$\int_0^\infty e^{-pt} f(t)\,dt \approx \sum_{i=0}^{n-1} [A_i^{(n)}(p) f(i) + B_i^{(n)}(p) f'(i)]\,.$$

AUTHOR'S SUMMARY

22 [2.25, 4, 7].—F. W. OLVER, *Asymptotics and Special Functions,* Academic Press, Inc., New York, 1974, xvi + 572 pp., 24 cm. Price $39.50.

This is a very satisfactory book, which combines sound mathematical analysis with a pervading sense of realism and practicality that will make it an extremely useful volume for applications of mathematics involving second-order linear ordinary differential equations and the classical special functions. The author has been a well-known contributor to the asymptotic theory of such equations for over twenty years. He has worked on the computational as well as on the theoretical aspects of these problems. In his own research, as in this book, he emphasizes results that can be used to compute, be it with pencil and paper or on electronic machines.

So much is known on the asymptotic approximations to solutions of ordinary linear differential equations that no single book can do justice to this whole body of knowledge. The author has wisely limited himself to differential equations of order two and has omitted all theories that do not imply computational results. A very distinctive feature of this book–and also of the author's own work–is the emphasis on error estimates. Usable, realistic inequalities for the remainder in asymptotic expansions are rarely found in the literature. The author has developed a practical scheme for the derivation of such bounds and he applies it throughout the volume.

The mathematical prerequisites are kept simple: Undergraduate level courses in advanced calculus and complex variable theory, and a first course in ordinary differential equations should suffice. It is true, on the other hand, that the presentation becomes more condensed as the book progresses, and some analytic proofs are described so briefly that the reader has to put in quite a bit of thinking to supply the details. To the serious student of the subject, the many examples and the over 500 exercises will be welcome. The variety and interesting nature of the exercises is impressive.

The first seven chapters contain the essentials of the subject: The classical special functions, the basic properties of second-order linear differential equations, the nature of asymptotic series and the various techniques for obtaining them from integral representations as well as from formal expansions. Chapter 6 is probably the most distinctive section of the book. It describes the author's version of what is frequently called the WKB method, a name he sensibly avoids in favor of the historically more accurate one of Liouville-Green Approximation. As developed by the author, it becomes a very flexible asymptotic tool complete with a general formula for a bound on the remainder. The technique is presented in such a way that it applies to asymptotic problems with a large parameter as well as to large independent variables, to unbounded domains as well as to turning point problems. One price that has to be paid for this generality is some lack of motivation at the beginning. It is not clear to the uninformed why certain terms are treated as small with respect to others. However, as the technique is applied in chapter after chapter, eventually the motivation becomes quite transparent.

Most of the material in the later chapters is of a more specialized nature. It includes, among other things, the Euler-Maclaurin formula, refinements of the saddlepoint method, turning and other transition points, and asymptotic connection problems.

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23 [2.25, 4, 7].–F. W. OLVER, *Introduction to Asymptotics and Special Functions*, Student Edition, Academic Press, Inc., New York, 1974, xii + 297 pp., 24 cm. Price $10.00.

The first seven chapters of the above reviewed volume are well suited to form the